



# Revista de Finanças Aplicadas

ISSN 2176-8854

[WWW.FINANCASAPLICADAS.NET](http://WWW.FINANCASAPLICADAS.NET)

## VALUE-AT-RISK OF BRAZILIAN ETFS WITH EXTREME VALUE THEORY APPROACH

**Yuri Sampaio Maluf, UNB/DF**

Professor de Administração da UNB/DF.

Mestrando em Estatística pela UNB/DF.

yurimaluf@gmail.com

**Otávio Ribeiro Medeiros, UNB/DF**

Professor de Finanças da UNB/DF

Pós Doutor em Economia pela University of Southampton.

otavio@unb.br

## VALUE-AT-RISK OF BRAZILIAN ETFS WITH EXTREME VALUE THEORY APPROACH

### OBJETIVO

Investigar como os investidores no Brasil estão expostos aos riscos através dos Exchanged Traded Funds (ETF) comparado com seus respectivos índices.

### METODOLOGIA

Foram comparados os riscos do ETF de Ibovespa e do próprio Ibovespa, de mai09 a jul13. O nível de risco foi calculado com os métodos VAR (Value at Risk) e ES (Expected Shortfalls), empregando a técnica de bootstrap. Foram incorporadas as características de heterocedasticidade e de caudas pesadas típicas em séries financeiras, tendo um bom ajustamento com a distribuição de Pareto generalizada.

### RESULTADOS E CONCLUSÕES

Os dados apontam que o investimento indireto na carteira de mercado com o ETF do Ibovespa oferece mais risco aos agentes do que o investimento direto dos ativos que compõe a carteira do índice Ibovespa.

### IMPLICAÇÕES PRÁTICAS

O maior risco do ETF de Ibovespa em relação ao próprio índice representa um custo do fundo em relação ao investimento direto nas ações que compõem o índice. Esse custo associado ao risco adicional deve ser adicionado à taxa de administração cobrada pelos gestores e administradores do ETF.

### PALAVRAS-CHAVE

ETF, Valor em Risco, Teoria Valores Extremos.

## VALUE-AT-RISK OF BRAZILIAN ETFS WITH EXTREME VALUE THEORY APPROACH

### OBJECTIVE

Investigate how investors in Brazil are exposed to risks via Exchanged Traded Funds (ETF) compared with their respective indexes.

### METHODOLOGY

It compares the risks of Bovespa Index ETF and the Bovespa index from May09 to jul13. The level of risk was calculated with the VAR (Value at Risk) and ES (Expected shortfalls) methods, using the bootstrap technique. The characteristics of heteroskedasticity and fat tails that are typical in financial time series were incorporated, having a good fit with the generalized Pareto distribution.

### RESULTS AND CONCLUSIONS

The data indicate that the indirect investment in the market portfolio with ETF of Bovespa is more risky to the agents than the direct investment of assets comprising the portfolio of the Bovespa index.

### PRACTICAL IMPLICATIONS

The greatest risk of ETF relative to the Bovespa index itself represents a cost to the fund in relation to direct investment in stocks that comprise the index. This cost associated with additional risk should be added to the administration fee charged by managers and administrators of ETF.

### KEYWORDS

ETF, Value-at-Risk, Extreme Value Theory.

Investigate how investors in Brazil are exposed to risks via Exchanged Traded Funds (ETF) compared with their respective indexes.

## INTRODUCTION

Exchanged traded funds represents an important innovation across global financial markets since the first one was launched in the Canadian stock market in 1989 (GALLAGHER; SEGARA, 2004). Afterwards, in 1993, this type of fund was introduced in the American market (HUANG; LIN, 2011). Known as ETF, they behave as usual closed-end funds holding any kind of assets such as stocks, commodities and bonds. In general, they are linked to some specific index and are traded close to its net asset value (NAV) over the trading day. In Brazil, the first ETF was launched only in 2004 with IBrX-50 as its underlying index.

Since ETF has a portfolio, it offers the benefits of diversification with only one share. In principle, the ETF shares should have the same performance of their underlying index, which would imply the same level of risk. However, due to the secondary market it is possible for them to diverge. This fact leads ETF shares to significantly increase their volatility (ROMPOTIS, 2007). Cherry (2004) argues that if ETF shares become more volatile, arbitrage opportunities might emerge. Another possibility, in accordance with the efficient market hypothesis, is that ETF shares might exhibit higher returns than its benchmark. One way to measure ETF risks is by Value at Risk - VAR.

In accordance with Choudhry (2006), the adoption of VAR methodology as a risk measure by bank regulators was a milestone risk management. Roughly speaking, VAR is an assessment of the amount of cash subject of loss. In more quantitative terms, VAR could be established as a high quantile of the returns distribution of some asset or portfolio. However, it has some drawbacks. In order to avoid it, works as Artzner et al (1999) suggest employing Expected

Shortfalls (ES) as an alternative risk measure. ES is the mean of losses when a given threshold is achieved. Either VAR or ES can be applied to assess the volatilities of ETFs and their underlying indexes which are then compared.

Within the parametric classes, there are some VAR implementation techniques. These models are based on some probability distribution associated with returns of some asset. The most widely used method is based on the Gaussian distribution. However, as stylized facts in finance indicate, asset returns exhibit heavy tails, so that applying the Gaussian distribution could bring some inconvenience. To some extent, this can be worse in the context of VAR, since it is focused on high quantiles. Another stylized fact highlighted by Cont (2001) is the presence of the heteroscedasticity in financial time series. As an alternative, a semi-parametric approach can be used to give some structure exclusively on tails and not on the entire distribution.

Considering these problems, McNeil and Frey (2000) propose the usage of Extreme Value Theory (EVT) which incorporates time-based volatility evolution to assess VAR. This method models tails of returns distributions that exceed a given threshold by Generalized Pareto distribution (GPD). As well as some VAR's methodologies, the McNeil and Frey (2000) approach produces a dynamic measure, that is, each period a new VAR is computed. This method also incorporates the presence of heteroscedasticity in the series, by applying GARCH models to decluster volatility.

Once VAR is estimated, it is possible to compare the volatilities of an ETF and its underlying index over time. Hence, in this paper, we analyze the existence of a risk differential between the main Brazilian Index – (Ibovespa) and its ETF –

(Ibovespa's iShare). Then, we also investigate whether their performances are the same. Rather than verifying only one realization process, we use a nonparametric bootstrap technique for time series to avoid data snooping effects, making it possible to test the hypothesis of no existence of difference in performance between them.

This paper is structured as follows. The second and third section explains relevant aspects and reviews some studies on ETFs and Value-at-Risk respectively. The following section covers the VAR approach and bootstrap methods in more detail. The fifth section presents the empirical results of VAR estimation, the adjustment by GPD and then compares their relative performance by means of bootstrapping. At the end we conclude.

## **EXCHANGED TRADED FUNDS**

Exchanged Traded Funds combines usual index funds with stocks. The main objective of an ETF, as any index fund, is to track some index, taken as benchmark. Its portfolio follows a passive investment strategy which holds the same securities basket of its underlying index. ETF funds can hold commodities, metals, stocks, bonds, currencies all included in only one share. Differently from open-end index funds, ETFs have the possibility to trade shares on stock exchanges, much like any stock throughout the trading day, implying more liquidity to investors (GASTINEAU, 2001). As a consequence, the value of an ETF tends to be close to its net asset value (NAV).

This kind of fund offers some advantages to investors. One of them is cost reduction. According to Fry (2008) in the U.S. market, ETF carries low management fees, ranging from 0.10% to 0.40%, while active mutual funds charge

from 1.00% to 2.00%. In the Brazilian market, the Ibovespa's ishare fund, which has the Ibovespa index as its benchmark, charges 0.54%<sup>1</sup> On the other hand, ANBIMA (2011) shows that in 2010 equities funds charge 2.20% on average.

Another significant advantage of ETFs, highlighted by Guedj and Huang (2009), is the possibility of creation and redemption of shares. However, this can be made only with a very large basket of stocks or shares determined by regulation of funds administration. The fact that ETF shares are continuously traded during the day and NAV values are directly observed leads ETF share prices close to their fundamental values (ENGLE; SARKAR, 2009).

Moreover, according to Rompotis (2005) ETF offers the benefit of risk mitigation, since investors can hold a diversified portfolio with only one share. Another feature, cited by Wild (2007), is the possibility to profit with slump share prices. In other words, different from other funds, ETFs allows investors to take a short position. In this way, it is feasible to establish a portfolio hedge in the spot market. For instance, a manager could make it by borrowing ETF shares in proportion to its systemic risk and hold a long position on his portfolio. Hence, when stock prices go down, the manager profits from ETF compensating portfolio losses.

Nevertheless, ETF investment could display some drawbacks. The amount of transactions of ETF shares does not necessary reflects the NAV. The presence of the secondary market could imply disparity of values (ROMPOTIS, 2007). This can lead to divergence of performance between an ETF and its underlying index (SHIN; SOYDEMIR, 2010). The deviation between these assets could be calculated by the ra-

---

<sup>1</sup> Available at <http://br.iShares.com>

tio of ETF share prices and NAV. Discounts over time is denominated tracking error. Some authors like Malkiel (1977), Frino and Gallagher (2011) and Chiang (1998) describe some aspects of tracking errors. A significant one stems from costs such as managerial fees, tax liabilities and brokerages. Another aspect is the liquidity of shares induced by bid-ask spreads. ETF returns incorporate dividends of securities contained in the portfolios. Finally, changing index composition and liquidity of portfolio could also influence tracking.

According to Cherry (2004), an ETF has a systematic variation of tracking error. This author states that discounts exhibit more time variation than expected and argues that ETFs are more volatile than its index. However, this implies that discount variations can predict future returns, but it is a prohibited efficiency market hypothesis, otherwise one can obtain abnormal return. Cherry (2004) explains excess volatility in terms of bid-ask spreads, dividends, interest rates and volume. From these reasons, it emerged in literature some studies that investigate dynamic of ETF in several markets.

In Asian markets Jares and Lavin (2004), Lin, Chan and Hsu (2005) and Wang, Liao and Yang (2009) have looked at the relationship between ETF shares and their underlying index. The first one investigates the pricing efficiency when asynchronous time trade of foreign ETFs and their benchmark exists. The last two studies examined the tracking error and the direction causality between an ETF and its respective index. Similarly in U.S market Dolvin (2002) and Marshall, Nguyen and Visaltanachoti (2010) both found some opportunities of arbitrage in the U.S market between ETF (SPY and IVV) of S&P500. Nonetheless, Elton, Gruber and Comer (2002) show that, in spite of the existence of de-



viations between SPY and its NAV, they vanish quickly. Simon and Sternberg (2005) provide an investigation about overreaction in European ETF markets in respect of their underlying index traded in the U.S market. Gallagher and Segara (2004) explore the Australian ETF's performance and conclude by pointing out to efficient ETF pricing. Finally, in the Brazilian market Maluf and Albuquerque (2013) investigate the possibility of arbitrage in high frequency data on the main Brazilian ETF. They found that after data-snooping filtering there is no arbitrage caused by disparity of tracking errors. Borges, Eid Junior and Yoshinaga (2012) analyze the tracking error of some Brazilian ETFs and compare them with open-end index funds. The results indicate that ETFs present better performance due to management fees and brokerage. Despite performance, open-end index funds track their underlying index closer than ETF.

Exchange Traded funds also provide an opportunity to diversify internationally since direct investment in foreign market is not always feasible. In this direction, Huang and Lin (2011) investigate the performance of ETFs, focusing on performance analysis, applying Dowd's method. This technique is similar to the Sharpe ratio but changing standard deviation to Value-at-Risk. They found that direct and indirect foreign investment provides the same performance after adjusting risk, i.e., - investors could obtain equivalent results by diversifying portfolios via ETF.

## **VALUE-AT-RISK AND EXPECTED SHORTFALLS**

The volatility of financial markets can produce wide effects in economy as a whole. Since the first Basle Agreement was established, the role of financial risk management has increased in importance (POON; GRANGER, 2003). Despite

being desirable, risk management does not exist to completely exclude the chance of losses. It is designed to control, assess the extent of exposure and minimize risk as much as possible. In any case, the measurement of volatility becomes an essential issue for many financial institutions. Banks and trading houses, in particular, are responsible for measuring risk exposure and then report it to their shareholders (CHOUDHRY, 2006).

In this regard, Value-at-Risk or simply VAR plays an important role. VAR can be defined as an assessed amount of the maximum loss which can occur with  $\alpha\%$  confidence over a given period. Essentially, VAR summarizes the market risk that some institution has with respect to its assets. So, this instrument is important to traders, risk managers and other market players when they are assessing risk exposure. By using VAR, shareholders and managers can decide whether they feel comfortable with a certain degree of risk (JORION, 1996).

The use of VAR was adopted by Basle Committee on Banking Supervision as a proposal to banks and financial market participants. The main goal was to ensure a minimum degree of prudence, transparency and consistency of capital requirement among banks. The Committee requires the daily measurement of VAR, regarding one tail with 99% confidence over a period of 10 trading days and the application of a historical data approach with the horizon of at least one year. Based on this, market players become able to speak a common language with respect to risk (BASLE COMMITTEE, 1996).

There are some approaches involving VAR estimation. The simplest and most widely used is based on historical data. This method consists in adopting one realization process

with length  $L$  and then setting a window size,  $W$ , to establish overlapping process subsamples,  $L-W+1$ . For each subsample  $\{r_1, \dots, r_w\}, \dots, \{r_{L-w+1}, \dots, r_L\}$  an empirical cdf<sup>2</sup> of returns is set and then used to assess  $\text{VAR}_t$ . Therefore, historical data does not require any assumption about the distribution of returns. This non-parametric approach relies on iid<sup>3</sup> process assumption. However, this assumption is not verified in empirical case.

Another approach is based on parametric models. These models such as *RiskMetrics* and GARCH incorporate more feasible features of financial series than the historical data approach. However, the cost of parametric models is the requirement of assumptions that are sometimes overly strong (JONDEAU; POON; ROCKINGER, 2007).

The main difficulty of these approaches lies with their premises. Non-parametric methods assume iid processes. In counterpart, the parametric case makes several assumptions for the whole distribution. An alternative method is the semi-parametric VAR approach which is designed to cope with these assumptions.

The risk estimation, in the sense of VAR, looks into the extreme events. So that, the semi-parametric VAR approaches makes assumptions exclusively on tails. This can be done with Extreme Value Theory (EVT).

Even being a widely used method of risk assessment, VAR has some drawbacks as an instrument for risk measurement. Artzner et al (1999) indicate the usage of so-called Expected Shortfalls (ES). The ES is the expected value of portfolio losses given some extent of losses. So its use is

---

<sup>2</sup> Cumulative Distribution Function

<sup>3</sup> Independent and Identically Distributed

made on the left tail of the distribution. ES explicitly incorporate all extreme losses. This measure also exhibits another advantage for portfolio selection optimization.

Risk analysis has an especial role in emerging countries. Due to their features, several studies have recently emerged and some of them use the EVT approach. Dimitrakopoulos, Kavussanos and Spyrou (2010) apply VAR methods in emerging equity markets, including Brazil. They observe that the VAR approach based on extreme value theory presents a good fit in backing tests. Another point highlighted by the authors is the reduction in the forecasting performance of VAR in periods of crisis compared with developed countries.

Gencay and Selcuk (2004) also investigate several VAR methods, including the Extreme Value Theory approach, in emerging markets. They conclude that EVT offers a useful tool for risk management, especially in these countries. In the Brazilian market, the authors show that EVT presents good adjustments in contrast with other methods, particularly on the left tail.

Ho et al (2000) analyze VAR on indexes, focused on Asian markets with Variance-Covariance and extreme value approaches. The empirical results show that the two methods are substantially different from each other and the maximum return distribution could be described EVT.

Sirr, Garvey and Gallagher (2011) also examine emerging countries, applying value-at-risk decomposition technique. The study is done from the perspective of European equity investors, which are exposed to equity and exchange rate risks. The results indicate that in Brazil and Mexico foreign

exchange represents a significant risk factor compared with other emerging countries.

The majority of recent papers points towards a positive performance of the EVT approach. These results reinforce the strength of this method when compared with others. As such studies, we considered a heavy tail with EVT, more specifically using Generalize Pareto Distribution (GPD), but also incorporating the heteroscedastic characteristic of financial time series.

## METHODOLOGY

In order to analyze the risk differential between ETF and its underlying index we first estimate VAR and ES for both assets and then compare these measures. Stylized facts in finance claim that returns tend to exhibit volatility clusters and that, even correcting for this feature residuals will still present heavy tails (CONT, 2001). Hence, it is necessary to verify these features and incorporate them in our modeling.

In this paper, we assess VAR and ES on the Ibovespa index and its ETF, as proposed by McNeil and Frey (2000). As mentioned previously, since an ETF is intended to reproduce its underlying index, it is expected they will have the same performance. Actually, this ETF could present more volatility than its underlying index (CHERRY, 2004). Therefore, we also compute the Sharpe ratio, suggested by Dowd (1999), to compare their performances adjusted by risk.

Afterwards, in order to avoid data snooping problems we apply a bootstrap technique for times series. This method does not require the assumption of any probabilistic distribution. More specifically, we constructed a bootstrap with blocks incorporating time series dependence. By applying

It compares the risks of Bovespa Index ETF and the Bovespa index from May09 to jul13. The level of risk was calculated with the VAR (Value at Risk) and ES (Expected shortfalls) methods, using the bootstrap technique. The characteristics of heteroskedasticity and fat tails that are typical in financial time series were incorporated, having a good fit with the generalized Pareto distribution.

this method, it is possible to investigate whether the risk differential between these assets are statistically significant.

### Methodology - Tail risk measure with EVT

As mentioned earlier, the VAR tail risk can be viewed as a quantile corresponding to the minimal potential loss that some asset could suffer in  $\alpha$  percent of worst cases, in a given time horizon. More formally we assume,

$$Var_{\theta,t} = -sup\{x: P(r_t \leq x|F_t) \leq \theta\} = -F_{R,t}^{-1}(\theta) \quad (1)$$

where  $F$  is the cdf of the return distribution of some assets at time  $t$ . Another measure of risk computed here is Expected Shortfalls. Also commented in a previous section, this measure assesses the mean loss given that it exceeds VAR. Thus ES is defined by,

$$ES_{\theta,t} = \int_{-\infty}^{\infty} I(r_t)_{[Var_{\theta,t}]} r_t f(r_t) dr_t, \quad (2)$$

being  $f(r_t)$  the density probability function of  $r_t$ , and  $I(r_t)_{[Var_{\theta,t}]}$  is the indicator function. The equation 12 results in the mathematical expectation  $-E[R_t | R_t \leq -Var_{\theta,t}]$ .

Jondeau et al (2007), argue that to compute VAR and ES four elements are necessary: the probability  $\alpha$ , the time horizon, the value of the portfolio at time  $t$  and finally the cdf of asset returns. The former is the main task of VAR assessment and the others points are given by practitioners.

Since value-at-risk is a high quantile, the focus is on the tails distribution  $f(r)$ <sup>4</sup>. EVT provides an approach to model the tails where extreme realizations occur. One way to do

---

<sup>4</sup>  $f(r) = \frac{\partial F(r)}{\partial r}$

this is by computing the values that exceed some high threshold  $u$  hence so-called *Peak-Over-Threshold* Jondeau et al (2007). These exceeding values can be modeled into another distribution, given by

$$\begin{aligned}
 F_u(y) &= \frac{P(R-u \leq y, R > u)}{P(R > u)} \\
 &= \frac{F_R(y+u) - F_R(u)}{1 - F_R(u)}
 \end{aligned}
 \tag{3}$$

considering  $R \leq \infty$  the upper endpoint of  $F_R$ , and  $r = y + u$ , for  $R > u$ , which corresponds to the upper tail of some asset return when multiplied by  $-1$ . Pickands (1975) demonstrates that  $F_u(y)$  could be well approximated by the Generalized Pareto Distribution (GPD) which has a cdf given by,

$$G(y; \xi, \psi) = \begin{cases} 1 - \left(1 + \frac{\xi}{\psi} y\right)^{-\frac{1}{\xi}}, & \text{if } \xi \neq 0, \\ 1 - \exp\left(-\frac{y}{\psi}\right), & \text{if } \xi = 0, \end{cases}
 \tag{4}$$

where  $\psi > 0$  and  $\xi$  are known as tail-indexes. When  $\xi \geq 0$  the support of  $G(y; \xi, \psi)$  is  $0 \leq y \leq \infty$ , otherwise  $y$  has an endpoint with  $0 \leq y \leq -\frac{1}{\xi}$ . This approach gives an opportunity to make

assumptions with respect to tails distributions only. That is, the focus is on tails and not on entire distribution which is irrelevant in a VAR context. Moreover, the parameter  $\xi$  identifies the tails shape.

Depending on  $\xi$  values there are three classes. When  $\xi < 0$ , the distribution has a finite endpoint, for instance, the beta distribution. The second is  $\xi = 0$  where the tail has an exponential decay, such as the normal, the exponential and

the gamma distributions. The last one corresponds to fat-tailed distributions implying power function decays in its tails such as the t-Student, Cauchy, Stable (with exponent smaller than 2) and Pareto distributions (JOUNDEAU, 2007).

Smith (1987) proposes an estimation of a tail index based on approximating  $F_u$  by GPD. Given a realization  $r_1, \dots, r_N$  a high threshold  $u$  is set and then the variable  $Y = R - u$ . Conditioning  $n$  exceeding values, the variable  $y$  is iid with distribution,

$$\begin{aligned} F(R) &= (1 - F(u))F_u(y - u) + F(u), \\ &= (1 - F(u))G_{\xi, \psi, u}(R) + F(u) \end{aligned} \tag{5}$$

for  $R > u$ . Then considering  $\frac{n}{N}$  as an estimator of  $F(u)$  we can apply the maximum-likelihood estimation function,

$$F(R) = 1 - N^{-1}n \left(1 + \frac{\hat{\xi}}{\hat{\psi}} y\right)^{-\frac{1}{\hat{\xi}}}. \tag{6}$$

However, iid realizations stemming from financial times series are not usual (CONT, 2001). McNeil and Frey (2000) suggest another procedure to overcome this problem, which is made in two steps. First, from the series returns an AR(1)-GARCH(1,1) model is estimated:

$$\begin{aligned} R_t^{ETF} &= \mu + \phi R_{t-1}^{ETF} + \delta_t^2, \\ \sigma_t^2 &= \omega + \alpha \delta_{t-1}^2 + \beta \sigma_{t-1}^2, \end{aligned} \tag{7}$$



where  $\hat{\sigma}_t = r_t - \hat{\mu}_t$  and  $\sigma_t$  is the conditional standard deviation. The residuals,  $z_t$  generated by this model, are computed as,

$$(z_{t-n+1}, \dots, z_t) = \left( \frac{r_{t-n+1} - \hat{\mu}_{t-n+1}}{\hat{\sigma}_{t-n+1}}, \dots, \frac{r_t - \hat{\mu}_t}{\hat{\sigma}_t} \right), \quad (8)$$

and if it was correctly specified,  $z_t$  should be iid. Assuming that we obtained an iid series the second step is to fit  $z_t$  by GPD. After estimating  $\hat{\xi}$  and  $\hat{\psi}$  it is possible to evaluate the features of the tail distribution.

In order to verify  $VAR_{\theta,t}$  it is necessary to calculate the  $\theta$ -quantile of  $z_t$  and then to transform it into  $R_t$ . Inverting the cdf of  $z_t$ , we obtain the  $\theta$ -quantile

$$z_{\theta} = \begin{cases} u + \frac{\hat{\psi}}{\hat{\xi}} \left( \left( \frac{N}{n} \theta \right)^{-\hat{\xi}} - 1 \right), & \text{if } \xi \neq 0, \\ u + \hat{\psi} \log \left( \frac{N}{n} \theta \right), & \text{if } \xi = 0, \end{cases} \quad (9)$$

From equation 1.7 we compute the dynamic  $VAR_{\theta,t}$  and  $ES_{\theta,t}$

$$VAR_{\theta,t} = -(\mu_t + z_{\theta} \sigma_t), \quad (10)$$

and then as consequence, we can compute,

$$ES_{\theta,t} = \frac{VAR_{\theta,t} + \hat{\psi} - \hat{\xi} z_{\theta}}{1 - \hat{\xi}}. \quad (11)$$

Due to the heteroscedasticity of financial time series McNeil and Frey (2000) point out that the unconditional EVT fre-

quently exhibits poor adjustment when compared with methods which incorporate this characteristic. This occurs because unconditional EVT cannot quickly react from volatility changes. For this reason, Conditional EVT seems to be more accurate than unconditional.

Despite the discussion on the tail risk measurement we would have to take into account the relationship between risk and return associated with asset evaluation. In this context, Dowd (1999) introduced a modified Sharpe ratio by changing standard deviation by VAR. This is applicable since VAR can be interpreted as a risk measure as well as standard deviation. Therefore, the performance measure with Dowd's approach is computed as,

$$SRD_t = \frac{R_t}{VAR_{\theta,t}}. \quad (12)$$

We shall use this measure to compare and test whether ETF is capable of replicating the performance of its underlying index. Nevertheless, there is only one realization produced by this technique. Hence a bootstrap method is used to test this difference.

### **Methodology - Bootstrap**

The series obtained from VAR, ES and SRD are generated through only one realization. It is possible to capture a specific series that brings evidence of differential risk and performance between ETF and its correspondent index when it does not actually occur. This phenomenon is denominated data-snooping effect. To avoid this kind of problem we apply a bootstrap technique.

The bootstrap was conceived as a method of estimation standard error of some a specific estimator  $\hat{\theta} = S(\mathbf{x})$  and do

not require any theoretical distribution assumptions from  $\mathbf{x}$  (EFRON; TIBSHIRANI, 1993). In such a way it is possible estimate the standard error of risk measures and performance. Therefore, this method allow us to construct a confidence interval for these measures and then to verify the significance of these differences.

The bootstrap sample consists of select B independent random resamples drawn from the empirical distribution  $\hat{F}(\mathbf{x})$  with replacement and the same length,

$$\mathbf{x}_b^* = (x_1^*, x_2^*, \dots, x_n^*). \quad (13)$$

After that, the estimator of statistic  $\hat{\theta}^*(b) = S(\mathbf{x}_b^*)$  is applied, for each of B bootstrap samples. The standard error is estimated from the vector,

$$(\hat{\theta}^*(1), \hat{\theta}^*(2), \dots, \hat{\theta}^*(B)) \quad (14)$$

and then calculated as,

$$se_B = \left\{ \sum_{b=1}^B [\hat{\theta}^*(b) - \hat{\theta}^*(\square)]^2 / (B-1) \right\}^{\frac{1}{2}} \quad (15)$$

where  $\hat{\theta}^*(\cdot) = \sum_{b=1}^B \hat{\theta}^*(2) / B$ .

The bootstrap considers iid samples. However, according to Cont (2001), it does not normally occur in financial times series. In order to overcome this problem a block bootstrap is employed. This method is applied in cases where dependent data exist. The blocks intend to keep a dependence structure within the data. The dependence is affected by the length of blocks. The longer the block, the stronger is dependence of process. On the other hand, the length of the sample decreases. Hence, there is an optimal choice of

block size that preserves dependence structure and at the same time does not lose data information.

Hall, Horowitz and Jing (1995) describe the conditions for choosing optimal nonoverlapping block sizes. The choice depends on the context, and in our case the block dimension should be  $l = N^{\frac{1}{5}}$ , where  $N$  is equal to the original sample size. This method is composed of blocks

$$B^* = (x_i, x_{i+1}, \dots, x_{i+l-1})^* \quad (16)$$

where  $i$  is the index of the series. We then construct a new series consisting of

$$S^* = \{B_1^*, B_2^*, \dots, B_K^*\}, \quad (17)$$

where  $K = \frac{N}{l}$  is the number of blocks.

In this paper we applied the block bootstrap to estimate the standard error of the difference of means of  $VAR^{ETF} - VAR^{NAV}$ ,  $ES^{ETF} - ES^{NAV}$  and  $SRD^{ETF} - SRD^{NAV}$ . Hence, we could verify whether ETF provide an effective replication. If it does not, what is the difference in their performance? In addition, it is necessary to check the confidence interval. We used, here, a  $BC_a$  method that offers an effectiveness form to construct the upper and lower boundaries. The  $BC_a$  method is based on the usual technique of percentiles, which use quantiles of observed  $\hat{\theta}^*(b)$  values. The percentiles interval establishes endpoints that cover  $1 - 2\alpha\%$ . It is given by,

$$\left[ \hat{\theta}_{lower}^{*(\alpha)}, \hat{\theta}_{upper}^{*(1-\alpha)} \right] = \left[ \hat{G}^{-1}(\alpha), \hat{G}^{-1}(1-\alpha) \right], \quad (18)$$

where  $\hat{G}^{-1}(\alpha)$  is the inverse cumulative distribution function of  $\hat{\theta}^*(b)$ .

The  $BC_a$  interval endpoint is similar to the percentile but is compounded with another two factors, acceleration  $\hat{a}$  and bias-correction  $\hat{z}_0$  that correct certain deficiencies of traditional percentile technique (EFRON; TIBSHIRANI, 1993). There are several forms to calculate  $\hat{a}$ , an expression exposed by these authors establishing acceleration as

$$\hat{a} = \frac{\sum_{i=1}^n (\hat{\theta}_{(i)} - \hat{\theta}_{(i-1)})^3}{6 \left\{ \sum_{i=1}^n (\hat{\theta}_{(i)} - \hat{\theta}_{(i-1)})^2 \right\}^{\frac{3}{2}}}, \quad (19)$$

defining  $\hat{\theta}_{(i)} = S(\mathbf{x}_{(i)})$  where  $\mathbf{x}_i$  is the original sample without the  $i$ -th element and  $\hat{\theta}_{(i)} = \sum_{i=1}^n \hat{\theta}_i / n$ . The amount  $\hat{a}$  is described as the rate of change of the standard error of  $\hat{\theta}$ . The quantity of bias-correction,  $\hat{z}_0$  is calculated as the proportion of the bootstrap replication inferior to the  $\hat{\theta}$

$$\hat{z}_0 = \Phi^{-1} \left( \frac{\#\{\hat{\theta}^*(b) < \hat{\theta}\}}{B} \right), \quad (20)$$

being  $\Phi^{-1}(\cdot)$  the inverse of the standard normal cumulative distribution function. Once computed  $\hat{a}$  and  $\hat{z}_0$  the next step is to construct endpoints of the intervals. The  $BC_a$  method aimed to cover  $1 - 2\alpha$  is defined as,

$$\left[ \hat{\theta}_{lower}^{BC_a}, \hat{\theta}_{upper}^{BC_a} \right] = \left[ \hat{\theta}^{*(\alpha_1)}, \hat{\theta}^{*(\alpha_2)} \right], \quad (21)$$

where

$$\alpha_1 = \Phi\left(\hat{z}_0 + \frac{\hat{z}_0 + z^{(\alpha)}}{1 - \hat{a}(\hat{z}_0 + z^{(\alpha)})}\right),$$

$$\alpha_2 = \Phi\left(\hat{z}_0 + \frac{\hat{z}_0 + z^{(1-\alpha)}}{1 - \hat{a}(\hat{z}_0 + z^{(1-\alpha)})}\right).$$
(22)

The value  $z^\alpha$  is the  $100\alpha$ -th percentile point of standard normal distribution and again  $\Phi(\cdot)$  represents the cumulative distribution function of standard normal. Notice that if the acceleration  $\hat{a}$  and the bias-correction  $\hat{z}_0$  are equal to zero then the endpoints of the confidence interval will coincide with the percentile method.

In this way, the bootstrap allows us to establish confidence intervals of  $\overline{VAR}^{ETF} - \overline{VAR}^{NAV}$ ,  $\overline{ES}^{ETF} - \overline{ES}^{NAV}$  and  $\overline{SRD}^{ETF} - \overline{SRD}^{NAV}$ . Thus it is possible to test whether ETF and its underlying index have the same risk and performance.

## EMPIRICAL ANALYSIS

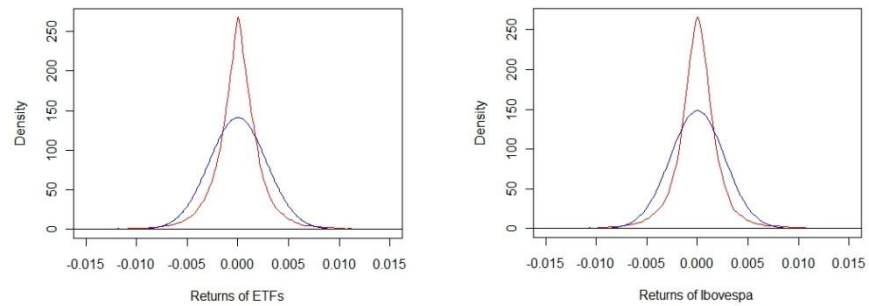
The sample used here was extracted from CMA system data base (Consultoria, Métodos, Acessoria e Mercantil S/A). A total of 22,255 observations, from 05/04/2009 to 07/13/2012 at a 15 minutes frequency trade were collected, with the prices negotiated at the After-Market being ignored. The time series consist of ETF iShare Ibovespa closing prices, available at the BOVA11 code, as well the Ibovespa index with its corresponding points. The missing values were linearly interpolated.

We started looking at the ETF return distribution and its underlying index. As can be seen in graphic Figure 1 both assets exhibit low tail decay when compared with the normal distribution. We also checked the existence of unit

The data indicate that the indirect investment in the market portfolio with ETF of Bovespa is more risky to the agents than the direct investment of assets comprising the portfolio of the Bovespa index.

roots for the return series. Using the usual ADF test the nonstationary null hypothesis was rejected at  $10^{-4}$  significance level, for these pair of times series.

**Figure 1: Kernel Estimation and Normal Distribution of ETF and Ibovespa returns**



This behavior suggests that, in such frequency, returns tend to exhibit heavy tails, as a consequence of volatility clusters. Applying the White test homoscedasticity is rejected in favor of heteroscedasticity a confidence level greater than  $10^{-4}$ . Initially we applied an AR(1)-GARCH(1,1) as declustering method. The results for both assets are shown in table Table 1 and Table 2.

**Table 1: AR(1)-GARCH(1,1) Estimation ETF**

	Coefficient	Std Error	z-Estimate	p-Value
$\hat{\mu}$	$5.98 \times 10^{-7}$	$1.54 \times 10^{-5}$	0.039	0.96
$\hat{\phi}$	$-2.99 \times 10^{-2}$	$9.00 \times 10^{-3}$	-3.319	0.000*
$\hat{\omega}$	$2.38 \times 10^{-6}$	$8.67 \times 10^{-8}$	27.524	0.000*
$\hat{\alpha}$	$4.01 \times 10^{-1}$	$1.55 \times 10^{-2}$	25.775	0.000*
$\hat{\beta}$	$3.94 \times 10^{-1}$	$1.51 \times 10^{-2}$	26.127	0.000*

The estimated parameters were computed by a quasi-maximum likelihood method and their standard errors were based on the Fisher information matrix. Except by  $\hat{\mu}_{ETF}$ ,  $\hat{\mu}_{Ibov}$

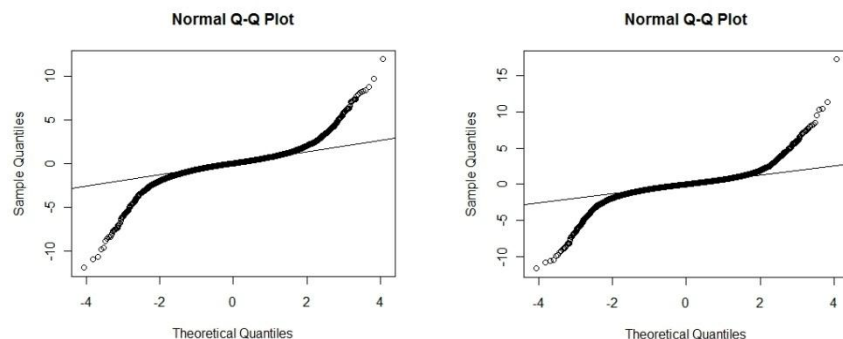
and  $\hat{\phi}_{Ibov}$  all of them are statistically significant, and  $\alpha + \beta < 1$  in both cases. The symbol (\*) means that the p-values are less than  $10^{-4}$ .

**Table 2: AR(1)-GARCH(1,1) Estimation Ibovespa**

	Coefficient	Std Error	z-Estatistic	p-Value
$\hat{\mu}$	$-2.06 \times 10^{-5}$	$1.54 \times 10^{-5}$	-1.339	0.181
$\hat{\phi}$	$9.12 \times 10^{-3}$	$9.51 \times 10^{-3}$	1.038	0.299
$\hat{\omega}$	$1.98 \times 10^{-6}$	$6.97 \times 10^{-8}$	28.381	0.000*
$\hat{\alpha}$	$3.43 \times 10^{-1}$	$1.52 \times 10^{-2}$	22.576	0.000*
$\hat{\beta}$	$4.80 \times 10^{-1}$	$1.30 \times 10^{-2}$	37.166	0.000*

After that, it is plausible to test if residuals are an iid series with normal distribution. For the first one we applied the Ljung-Box test for ETF and Ibovespa residuals series, and both were not statistically significant with 0.60 and 0.29 p-values respectively. Finally, in order to verify the adjustment of residuals to the normal distribution we employed a QQ-plot. The results are exhibited below.

**Figure 2: QQ Plot of ETF and Ibovespa returns**





Even applying a GARCH model, the residuals still suggest heavy tails as we can see in the Figure 2. This result is not different from stylized facts in finance commented by (CONT, 2001). However, our focus is on extreme occurrences where a risk asset manager is concerned.

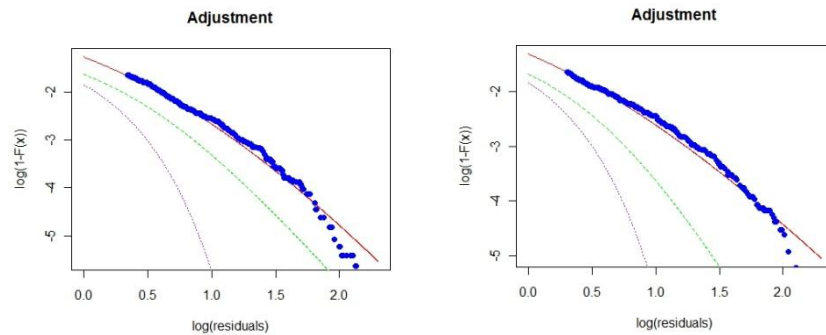
Modeling loss tails of ETF and Ibovespa index residuals with GPD exposed in equation 1.6 we obtain the tail index. The parameters were obtained through maximum likelihood estimation and the standard error was based again on the Fisher information matrix. The results are presented in Table 3.

**Table 3: Parameters Estimation of Generalized Pareto Distribution**

ETF				
	Coefficient	Std Error	z-Estatistic	p-Value
$\hat{\xi}$	0.323	0.041	7.87	0.000*
$\hat{\psi}$	0.634	0.0317	20.00	0.000*
Ibovespa				
	Coefficient	Std Error	z-Estatistic	p-Value
$\hat{\xi}$	0.41	0.044	9.53	0.000*
$\hat{\psi}$	0.57	0.029	19.65	0.000*

Corroborating with Figure 2 the tails indexes were  $\xi > 0$  for all assets, which reinforce the evidence of heavy tails. That is, by applying some VAR methodology that consider normal or even t-Student distributions will underestimate the risk. This can be seen from Figure 3.

**Figure 3: Adjustment of residuals from different distributions**



These graphics highlight the superiority of the data fit by GPD compared with others distributions. The Table 4 displays some summary statistics of VARs series computed by equation 11.

**Table 4: Summary Statistic of ETF and Ibovespa VAR**

	Sample Mean	Std Deviation	Max	Min
$Var_{ETF}$	0.0073	0.0032	0.06	0.004
$Var_{ibov}$	0.0069	0.0029	0.072	0.005

The results shown in Table 4 suggest some asymmetry of risk between ETF iShare Ibovespa and Ibovespa index. Therefore, at first look, indirect investment in Brazilian main assets index holds more risk than direct investment. Nevertheless, the results were estimated from only one realization of time series and as commented early this could bring some bias in our analysis.

Thus we performed 100,000 bootstraps replications with approximately 3,160 blocks each in order to analyze the significance of difference between the VARs of these assets. The bootstrap was designed to cover confidence intervals of 95% based on the  $BC_a$  method. The output is presented in the following Table 5.

<b>Table 5: Blocks Bootstrap VAR</b>					
	$\hat{\theta}^*(\cdot)$	B	$se_B$	$\hat{\theta}_{lower}^{BC_a}$	$\hat{\theta}_{upper}^{BC_a}$
$Var_{ETF} - Var_{ibov}$	$3.5 \times 10^{-4}$	100,000	$1.54 \times 10^{-5}$	$3.26 \times 10^{-4}$	$3.75 \times 10^{-4}$

Indeed, according to VAR bootstrap results, the ETF investors are subject to hold more risk than direct index investors. However, this kind of investors suffers a drawback since they incur in higher costs, since it is necessary to buy a couple of stocks instead of only one share.

Up to this point only VAR was considered as a risk measure. Another possibility is the Expected Shortfalls approach. However the results of the bootstrap method pointed to the same conclusion. That is, the investors who use ETF are subject to more risk. The Table 5 shows the results of 100,000 bootstrap replications. As we can see the difference between ES of direct and indirect investments are still statistically significant, even filtered by a data-snooping factor.

<b>Table 6: Blocks Bootstrap ES</b>					
	$\hat{\theta}^*(\cdot)$	B	$se_B$	$\hat{\theta}_{lower}^{BC_a}$	$\hat{\theta}_{upper}^{BC_a}$
$ES_{ETF} - ES_{ibov}$	$3.19 \times 10^{-4}$	100,000	$1.32 \times 10^{-5}$	$2.98 \times 10^{-4}$	$3.41 \times 10^{-4}$

The ETF analysis should include other perspectives such as performance. So it is important to compare ETF and Ibovespa index returns adjusted by some risk measure. We apply the Dowd (1999) method. Through the VAR series, it is created a Dowd series performance. In order to investigate the difference in performance between these assets we also use the bootstrap method.

At first glance, the performance of Ibovespa seems better than ETF. The  $SRD_{Ibov} = 67 \times 10^{-5}$  and  $SRD_{ETF} = 61 \times 10^{-5}$ , this outcome occurs because the risk of ETF overcomes the returns more than Ibovespa. However, when we observe the results of bootstrap of the Dowd series it is clear that the performance between these assets disappear.

**Table 7: Blocks Bootstrap SDR**

	$\hat{\theta}^*(\cdot)$	<b>B</b>	$se_B$	$\hat{\theta}_{lower}^{BC_a}$	$\hat{\theta}_{upper}^{BC_a}$
$SRD_{ETF} - SRD_{ibov}$	$86 \times 10^{-5}$	100,00 0	$67.5 \times 10^{-5}$	$-22.2 \times 10^{-5}$	$199 \times 10^{-5}$

In other words the better returns of ETF are priced by enhanced of risk. These results could be viewed above based on 100,000 bootstrap replications.

The greatest risk of ETF relative to the Bovespa index itself represents a cost to the fund in relation to direct investment in stocks that comprise the index. This cost associated with additional risk should be added to the administration fee charged by managers and administrators of ETF.

## CONCLUSION

In this paper we investigated the risk of the main exchange traded fund in Brazil Ishare Ibovespa and its benchmark index in high frequency data. As a measure of risk we adopted Value-at-Risk - VAR and Expected Shortfalls - ES. Afterwards, we carried out an analysis of their performance adjusted by risk. Thereby, our main goal was to compare these measures and verify whether ETF investors are subjected with the same risk of its index.

Brazil, as an emerging country, exhibits high volatility compared with other developed countries. Therefore, reinforced by stylized facts in finance, their assets have a fat tail on its returns distribution, particularly on the left side. This fact demands a different treatment of usual Gaussian return distribution. Again, stylized facts allege the presence of heteroscedasticity of variance. To cope with these problems we used a method, based on extreme value theory, proposed by (MCNEIL; FREY, 2000).

This method produces a dynamic measure of risk, that is, there is series of VAR. Thus, in order to analyze Ishare Ibovespa ETF and Ibovespa Index risks we compared their risk by each time. However, to avoid data-snooping effects we applied bootstrap for time series. The difference in performance between these assets was also calculated.

The estimation of the tail distribution, assessed by the generalized Pareto distribution, showed a good fit for both series when compared with the Gaussian distribution or even with the t-Student when the former has heavy tails. Moreover, the results pointed out that Ishare Ibovespa carry more risk than its benchmark. In other words, the agents who

make direct investment via Index are less exposed to volatility.

## REFERENCES

ANBIMA. (2011). Fundo de investimento: Relatório de taxas de administração. São Paulo: Associação Brasileira das Entidades dos Mercados Financeiros e de Capitais.

ARTZNER, P.; DELBAEN, F.; EBER, J. M.; HEATH, D.; Coherent measure of risk. *Mathematical Finance*, v. 9 , p. 203-228, 1999.

BASLECOMMITTEE. Overview of the amendment to the capital accord to incorporate market risks. *Bank of International Settlements*. 1996.

BORGES, E. C.; EID JR, W.; YOSHINAGA, C. E. Exchange traded funds versus fundos indexados no brasil. *Revista de Finanças Aplicadas*, v.1, p. 1 – 15, 2012.

CHERRY, J. The limits of arbitrage: Evidence from exchange traded funds. *Working Paper*, University of California-Berkeley, 2004.

CHIANG, W. Indexing for maximum investment results. *Working Paper*. Chicago, Illinois, USA, 1998.

CHOUDHRY, M. *An introduction to value at risk*. John Wiley, 2006.

CONT, R. Empirical properties of asset returns: stylized facts and statistical issues. *Quantitative Finance*, v.1, p.223-236, 2001.

DIMITRAKOPOULOS, D. N.; KAVUSSANOS, M. G.; SPYROU, S. I. Value at risk models for volatile emerging

markets equity portfolios. *The Quarterly Review of Economics and Finance*, v.50, p. 515-526, 2010.

DOLVIN, S. D. Etf: Arbitrage opportunities and market forecasting. *Journal of Index Investing*, v.1, p.107-116, 2009.

DOWD, K. A value at risk approach to risk-return analysis. *The Journal of Portfolio Management*, v.25, p.60-67, 1999.

EFRON, B.; TIBSHIRANI, R. J. *An introduction to the bootstrap*. CHAPMAN HALL/CRC, 1993.

ELTON, E.; GRUBER, M.; COMER, G. L. K. Spiders: Where are the bugs? *Journal of Business*, v.75, p.453-472, 2002.

ENGLE, R. F.; SARKAR, D. Pricing exchange traded funds. *Working Paper*, New York University, 2002.

Fry, D. H. *ETFs hedge funds*. Wiley Finance, 2008.

GALLAGHER, D. R.; SEGARA, R. The performance and trading characteristics of exchange-traded funds. *Working Paper*, The University of New South Wales, 2004.

GASTINEAU, G. L. An introduction to exchange-traded funds (etfs). *Journal of Portfolio Management and Economics*, v.27, p.88-96, 2001.

GENCAY, R.; SELCUK, F. Extreme value theory and value-at-risk relative performance in emerging markets. *International Journal of Forecasting*, v.20, p. 287-303, 2004.

GUEDJ, I.; HUANG, J. Are etfs replacing index mutual funds? *Working paper*, University of Texas, 2009.

HALL, P.; HOROWITZ, J. L.; JING, B. On blocking rules for bootstrap with dependent data. *Biometrika*, v.82, p.561-574, 1995.

HO, L. C.; BURRIDGE, P.; CADLE, J.; THEOBALD, M. Value-at-risk: Applying the extreme value approach to Asian markets in the recent financial turmoil. *Pacific-Basin Finance Journal*, v.8, p.249-275, 2000.

HUANG, M. Y.; LIN, J. B. Do etfs provide effective international diversification? *Research in International Business and Finance*, v.25, p.335-344, 2011.

JARES, T. E.; LAVIN, A. M. Japan and hong-kong exchange traded fund etfs: Discounts, return and trading strategies. *Journal of Financial Services Research*, v.25, p.57-69,2004.

JONDEAU, E.; POON, S.; ROCKINGER, M. *Financial modeling under non-gaussian distribution*. Springer, 2007.

JORION, P. Risk: Measuring the risk in value at risk. *Financial Analysts Journal*, v.52, p.47-56, 1996.

LIN, C.; CHAN, S.; HSU, H. Pricing efficiency of exchange traded funds in Taiwan. *Journal of Asset Management*, v.7, p.60-68, 2005.

MALKIEL, B. The valuation of closed-end investment company shares. *Journal of Finance*, v. 32, p.847-859, 1977.

MALUF, Y. S.; ALBUQUERQUE, P. H. M. Empirical evidence: Arbitrage with exchange-traded funds (etfs) on the brazilian market. *Revista de Contabilidade e Finanças*, v.24, p.64 – 74, 2013.



MARSHALL, B. R.; NGUYEN, N. H.; VISALTANACHOTI, N. Etf arbitrage. *Working Paper*, University of Auckland, New Zeland, 2010.

MCNEIL, A. J.; FREY, R. Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach. *Journal of Empirical Finance*, v.7, p.271-300, 2000.

PICKANDS, J. Statistical inference using extreme order statistics. *Annals of Statistics*, v.3, p.119-131, 1975.

POON, S. H.; GRANGER, C. W. J. Forecasting volatility in financial markets: A review. *Journal of Economic Literature*, v.41, p.478-539, 2003.

ROMPOTIS, G. An empirical comparing investigation on exchange traded funds and index fund performance. *Working Paper*, University of Athens Greece, 2005.

ROMPOTIS, G. An empirical look on exchange traded funds. *Working Paper*, University of Athens Greece, 2007.

SHIN, S.; SOYDEMIR, G. Exchange-traded funds, persistence in tracking errors and information dissemination. *Journal of Multinational Financial Management*, v.20, p.214 – 234, 2010.

SIMON, D. P.; STERNBERG, J. S. Overreaction and trading strategies in European ishare. *Journal of Alternative Investment*, v.8, p.29-41, 2005.

SIRR, G.; GARVEY, J.; GALLAGHER, L. Emerging markets and portfolio foreign exchange risk: An empirical investigation using a value-at-risk decomposition technique. *Journal of International Money and Finance*, v.30, p.1749- 1772, 2011.

SMITH, R. L. Estimating tails of probability distributions. *Annals of Statistics*, v.15, p.1174-1207, 1987.

WANG, C. C.; LIAU, Y. S.; YANG, J. J. W. Information spill-over in the spot and etf indices in Taiwan. *Global Journal of Business Research*, v.3, p.117-131, 2009.

WILD, R. *Exchange-traded funds for dummies*. Indianapolis, Indiana: Wiley Publishing, 2007.