Revista de Finanças Aplicadas

www.financasaplicadas.net
ISSN 2176-8854

AVALIAÇÃO DE PORTFÓLIO DAS ESTRATÉGIAS DE INVESTIMENTO DE TIMING DE VOLATILIDADE E TIMING DE RECOMPENSA AO RISCO: O CASO BRASILEIRO

PORTFOLIO EVALUATION OF VOLATILITY TIMING AND REWARD TO RISK TIMING INVESTMENT STRATEGIES: THE BRAZILIAN CASE

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Recebido em 20/01/2016
Aprovado em 29/03/2016
Disponibilizado 19/08/2016
Avaliado pelo sistema *double blind review*

AVALIAÇÃO DE PORTFÓLIO DE ESTRATÉGIAS DE INVESTIMENTO DE TIMING DE VOLATILIDADE E TIMING DE RECOMPENSA AO RISCO: O CASO BRASILEIRO

OBJETIVOS

A presente pesquisa teve como objetivo verificar a performance dos modelos de seleção de portfólio baseados em *Timing* de volatilidade (VT) e *Timing* de recompensa ao risco (RRT), comparando-os com o Portfólio Ingênuo e o de Média-Variância, aplicados ao mercado de capitais brasileiro.

METODOLOGIA

A metodologia utilizada consistiu em aplicar as estratégias de construção de portfólios VT, RRT, Ingênuo e Média-variância, considerando diferentes níveis de rebalanceamento destes. Os ativos utilizados na análise foram aqueles incluídos no índice Ibovespa no período de Janeiro de 2004 a Dezembro de 2014. Foram utilizados indicadores estatísticos e financeiros para mensurar a performance das estratégias, assim como medidas de *turnover* e custos de transação.

RESULTADOSS E CONCLUSÕES

Foi possível comparar as estratégias VT e RRT com as carteiras de variância mínima (Wvm), o Ibovespa (Ibov) e a carteira Ingênua. O Ibov e a carteira Ingênua apresentaram os menores retornos. Por outro lado, a Wvm, e as carteiras VT e RRT com altos níveis de agressividade dos investidores a favor de ativos menos voláteis ($\eta = 4$), tiveram o melhor desempenho (Índice de Sharpe mensal de 6,6%, 5,98% e 5,08%, respectivamente). Além disso, as carteiras VT4 e Wvm mantiveram consistentemente um *turnover* baixo. No entanto, as carteiras RRT apresentaram alta rotatividade dos ativos. Na análise por subperíodos, os resultados apontaram que a melhor escolha de portfólio depende do cenário econômico brasileiro. Durante o primeiro subperíodo a RRTbm4 teve os melhores resultados com 45,65% retorno anualizado e 223,7% de Índice Sharpe. No segundo subperíodo a RRTk4 ficou com um retorno anualizado de 21,33% e um Índice de Sharpe de 62,33%. No último subperíodo, no entanto, nenhuma das estratégias apresentou retornos ou índices de Sharpe positivos.

IMPLICAÇÕES PRÁTICAS

Esta pesquisa mostrou que a escolha da melhor estratégia de carteira depende da conjuntura econômica vigente no mercado brasileiro, já que algumas das estratégias tiveram melhores resultados em subperíodos específicos. No entanto, na maioria dos casos, carteiras que dão mais peso para os ativos menos voláteis, como a de variância mínima, de *timing* de volatilidade e de *timing* de recompensa pelo risco com $\eta = 4$, podem resultar em melhor desempenho (Índice de Sharpe), sem aumento significativo dos custos de transação.

PALAVRAS-CHAVE

Seleção de Portfólio, *timing* de volatilidade, *timing* de recompensa ao risco.

PORTFOLIO EVALUATION OF VOLATILITY TIMING AND REWARD TO RISK TIMING INVESTMENT STRATEGIES: THE BRAZILIAN CASE

OBJECTIVES

This research aimed to verify the performance of the Volatility Timing (VT) and Reward to Risk Timing (RRT) models of portfolio selection when compared with the Naïve and Mean-Variance ones, applied to the Brazilian stock market.

METHODOLOGY

The methodology consists in applying the VT, RRT, Naïve and the Mean-Variance portfolio strategies, considering different tuning levels of rebalancing portfolios. The assets employed in the analysis were those included in the Ibovespa Index in the period from January of 2004 through December of 2014. We used statistical and financial indicators to measure the performance of the strategies, and measure its turnover and transaction costs.

RESULTS AND CONCLUSIONS

It was possible to compare strategies against the Minimum Variance (Wvm) portfolio, the Ibovespa (Ibov) and the Naïve portfolios. The Ibov and Naïve presented the lowest portfolios returns. In the other the hand the Wvm, VT and RRT, with high investors aggressiveness in favor to less volatile assets (η=4), had the highest performance (monthly Sharpe Index 6.6%, 5.98% and 5.08%, respectively). In addition, the VT4 and the Wvm portfolios consistently preserved a low turnover. However, the RRT portfolios presented high turnover. Analyzed by sub-periods, the results pointed out that the best choice of portfolio depends upon the Brazilian economic scenario. During the first sub period RRTbm4 had the best results with 45.65% annualized return and a 223.7% Sharpe Ratio. In the second sub period the RRTk4 featured with a 21.33% annualized return and a Sharpe Ratio of 62.33%. In the last sub period, however, no strategies presented positive returns or Sharpe Ratios.

PRACTICAL ISSUES

This research evidenced that the best choice of portfolio strategies depends upon the economic setting that the Brazilian market is undergoing, because some of the strategies had better results in specific periods. However, in most cases, portfolios that give more weights to less volatile assets, such as minimum variance, and volatility timing and reward to risk timing with η=4, would produce better performance (Sharpe Ratio) without significant increase in transaction costs.

KEY WORDS

Portfolio selection, Volatility Timing, Reward to Risk Timing.

INTRODUCTION

Several changes had occurred in the modern finance theory, having Harry Markowitz as one of the first mark of these changes with the article "Portfolio Selection" in 1952. In that, the author realized that normal investors usually followed the rule of choosing the higher return and lower volatility assets, which could result in a risky allocation strategy (MARKOWITZ, 1952).

Assuming that risk is the quantification of the chance of happening something differently from the expected and return is the expected payoff on the risk of a chosen asset, Markowitz (1952) suggested a new rule of investment called diversification. In this case, the investor chooses several assets, creating a portfolio that tries to reduce the risk among the chosen assets. Those portfolios created following the Markowitz's diversification theory are called Mean-Variance (MV).

Even though the MV - focused on identifying the efficient frontier – had been used in many recent studies, many criticisms were made to this theory since its proposition. Tu and Zhou (2011), for example, claim that the results obtained by MV technique may contain estimation errors given the use of historical data in its calculations. To correct these errors, it has been created over time different methodologies using forms of optimization that can achieve minimum risk with higher returns, different from those from the MV approach, as we can see in DeMiguel et al. (2009) and Fletcher (2011).

On the other hand, Maillard, Roncalli and Teiletche (2008) believe that the market still has a large fraction of investors that prefer heuristic methods. Also the problem of estimation errors when forecasting returns by the unrestricted Mean-Variance approach, as pointed by Sharma (2015) or Medeiros, Passos and Vasconcelos (2014) led investors to use one exclusive case of the Mean-Variance method, the Minimum-Variance (Wvm) portfolio, thus focusing on volatility (risk) based strategy.

In contrast to those optimized portfolios, the Naïve – innocent portfolio-, distributes the investment equally among the N selected assets (TU, ZHOU, 2011). Several studies, such as Tu and Zhou (2011), DeMiguel et al. (2009), and Fletcher (2011) claim that despite not having a theory behind, the naïve portfolio performs well when compared to other portfolios, often being more interesting and desired.

To find high levels of the performance indicators, achieving very favorable returns and risks, authors increasingly used robust forms of portfolio optimization. For example, methods of *"shrinkage"* or tuning-parameters as in Ledoit and Wolf (2003), Tu and Zhou (2011), and Kirby and Ostdiek (2012b) trying to reduce estimation errors. Despite finding some interesting results, some techniques also increase asset turnover. Therefore, the high turnover of those methods also brings high costs for their portfolios, thus made their strategies not desirable as expected.

Seeking to reduce such problems, Kirby and Ostdiek (2012a) created two different methods that use a combination of the Naïve and the Minimum-Variance portfolio, in which the weights of the assets are limited to never be negative: Volatility Timing (VT) and Reward to Risk (RRT). These two methods were tested in foreign markets, obtaining very good results for the investors. Because they do not need optimization, the estimation of the portfolio weights is simpler and faster compared to other approaches. Furthermore, the Kirby and Ostdiek (2012a) experiments showed that both methodologies had better performance indicators than the innocent portfolio, and also had low levels of turnover, hence low transaction costs.

As long as the stock markets have peculiarities, it is important to verify if the new methods can be applied and bring the same results in other countries. Based on that, this research aimed to verify the performance of the VT and RRT models proposed by Kirby and Ostdiek (2012a) compared with the Naïve and Minimum-variance ones applied to the Brazilian stock market.

The next section will present the theoretical framework; after that, the analytical framework will be presented; the results and discussions are shown on topic 4; at the last one, we will present conclusion of this research, its limitations and some suggestions for future research.

THEORETICAL FRAMEWORK

Markowitz revolutionized the field of finance and capital markets, publishing a world-renowned paper "Portfolio Selection" in 1952. The diversification rule proposed by Markowitz (1952) in that article aimed to eliminate the strategies that separated the expected return of an asset's risk. The author claimed that risk and return are extremely correlated traits in which if the asset has a high risk it is required by the investor a higher return on it.

> "The hypothesis (or maximum) that the investor would (or should) maximize the return must be rejected. If we ignore the imperfections of the market, this rule would mean that there is never a diversified portfolio that is preferred over those who are not diversified. Diversification is observed and sensitive; the rule of behavior that does not imply the superiority of diversification should be rejected, so how much chance as maximum." (Markowitz, 1952, p.77).

The possibility of risk reduction with the diversification turned the risk calculation (or estimation) into a very important issue in finance. Risk can be approximated by various ways, but variance and standard deviation are frequently used by many authors in their calculations. Markowitz as well realized in his studies the possible use of correlation and covariance to represent the relationship between the movements of the assets. Separate securities in the market yet have a relationship between their movements, which directly affect their volatilities. Those co-movements could be calculated and added to the formulations to achieve the portfolio risk. The covariance is given by:

$$\sigma_{ij} = E\{(R_i - \bar{R}_i) * (R_j - \bar{R}_j)\} \text{ or } \sigma_{ij} = \rho_{ij} * \sigma_i * \sigma_j \quad (1)$$

$$\text{and: } \rho_{ij} = \sigma_{ij} \div (\sigma_i * \sigma_j) \quad \text{where: } -1 \leq \rho_{ij} \leq 1 \quad (2)$$

Where σ_{ij} represent the covariance of an asset *i* with another asset *j*; R_i the expected return on asset *i*; $\bar{R}_i$ the average return of asset i; $(R_i - \bar{R}_i)$ the deviation of returns; ρ_{ij} the correlation between assets *i* and *j*; $\sigma_i$and σ_j are the standard deviations of assets *i* and *j* respectively.

In addition, Markowitz (1952) proposed that to diversify a portfolio, investor should not only add new assets, but should also make sure that the correlations between its assets were closest to -1 as possible, which means that two assets have perfect inverse correlation. In this case, if an asset reaches 10% return the other will have -10%, but if the correlation is equal to 1, if one asset had 10% return the other would have the same return. Santos e Tessari (2012) enforced the importance of this approach based on correlation and covariance affecting the relationship between risks and returns, eliminating the idea of independence between the risks of the assets within a portfolio.

From these considerations, Markowitz (1952) calculated the optimal weights for a portfolio that he would call efficient. To show this, he simplified his theory using only portfolios of three and four assets, and a stationary price process to calculate the historical returns

(SHARMA, 2015) to create a covariance matrix. The mathematical formula used to calculate the variance of a portfolio is:

$$\sigma_p^2 = X' * M * X \tag{3}$$

Where X has the portfolio vector weights, M is the matrix created from the covariance among assets in the portfolio.

To calculate the returns and find weights for the efficient portfolios, historical returns, or its excess over the risk free rate, and their arithmetic mean within a given timeline were used. The returns vector is then multiplied by the inverse of the covariance matrix to find the weight vector.

$$X = M^{-1} * \bar{r} \tag{4}$$

Where X is the weight vector; M^{-1} is the inverse covariance matrix, and $\bar{r}$ the vector of returns.

The efficient frontier of Markowitz (1952) is the connection of all efficient portfolios calculated using the proposed methods. Thus, combining the Minimum variance portfolio (MinV), which is found using a return vector of "ones" in equation 4, with the Tangency Portfolio (TP), that has excess returns as the return vector, given by $\bar{r} = \bar{R}_i - R_f$, where $\bar{R}_i$ is the average return of asset *i* and R_f is the return of a risk-free asset. This combination of the two portfolios is given by a *k* weighting in risky assets from the TP and (*1 -k*) weighting in the MinV).

The efficient frontier is found from two basic concepts in the literature and should be followed: (1) the investor will always prefer the portfolio with the highest possible return for a given level of risk, and (2) the investor always prefer the portfolio with minimum risk for a given level of return (Markowitz, 1952).

The theories of Markowitz (1952) became the foundation for the evolution of modern finance, but also arose several criticisms to his methods. Over time some research found empirical evidences that the author's investment strategy was less interesting than other methods, as presented by Tu and Zhou (2011), Kirby and Ostdiek (2012) among others.

Maillard et al. (2008) reinforce the idea that the MV is a very attractive strategy, but concentrate its portfolios in a few subsets of securities. In addition, this approach is over sensitive to input changes and therefore, the problems that Markowitz (1952) rules encountered, inspired many authors to use different approaches such as risk-based methods (Sharma, 2015).

According to Lee (2011) the financial crisis of 2008 made investors to question the theories in the construction of their portfolio. To diverge from the forecasting of returns, that is known to have great estimation errors some portfolios studies focused solely on the forecasting of the risk as an input (LEDOIT, WOLF, 2003).

Along the time, different kinds of strategies where proposed. One strategy that does not use means as an input is the Naïve portfolio. Duchin and Levy (2009) described it as a strategy that has been used by Babylonian Talmud. Tu and Zhou (2011) also claim that this strategy has around 1500 years old. It is determined by equal division of an amount to be invested among the assets chosen by the investor: $\frac{1}{N}$. Despite using a non-complex mathematical approach, the results are surprising when compared to other portfolios such as the unconstrained MV. The Naïve portfolio is often superior to other portfolios and when it does not, optimization methods of mean-variance portfolios were required to obtain better performance. Fletcher (2011) says that the shortcomings from practical implementation of the mean-variance analysis are the estimation of risks posed by the forecasts of the covariance matrix and returns on assets.

The theory of Markowitz (1952) helped in the understanding of several existing factors that should be analyzed prior to any investment in the capital market, but this did not solve the problems of the unpredictability of the market and consequently the estimation errors that come from the estimates of the covariance matrix and the expected returns. William Sharpe (1964) sought to eliminate some of the returns estimation problems, creating from the models of Markowitz (1952), a new model called the CAPM (Capital Asset Pricing Model).

This new model had only one factor, but was considered an evolution in estimation techniques of return of assets. Given by:

$$E(R_i) = R_f + \beta_i (E(R_m) - R_f) \quad (5)$$

$$\beta_i = \frac{Cov(i,m)}{Var(m)} \quad (6)$$

Where E (R_i) represent the expected return for the chosen asset; R_f the return on risk-free asset; β_i is the asset sensitivity to the market; E (R_m) the return expected by the market; Cov (i, m) the covariance between the return of the asset and the market one and Var (m) the variance of the market return.

The beta, at that moment, became a more accurate measure of risk and soon came to be used more frequently than the standard deviation proposed by Markowitz. Despite being a great model, the CAPM still had difficulties to express some information about other important characteristics of assets. Fama and French (1993) showed in their studies that in most cases factors such as size of the companies or the book-to-market ratio had better evidences that explained the sensitivity of the average returns of the securities. Carhart (1997) added the *momentum* factor to the CAPM used by Fama and French (1993) and obtained even better results in the average return estimation. The four-factor model Fama and French and Carhart (FFC) can be determined by:

$$R_i = R_f + \beta_i(R_m - R_f) + \beta_{SMB}(SMB) + \beta_{HML}(HML) + \beta_{WML}(WML) \quad (7)$$

On what: R_i is the expected return on the asset *i*; R_f the return of risk-free asset; R_m the return of the market portfolio; β_i is the beta sensitivity of the market portfolio and the portfolio *i*; β_{SMB} the beta that represents the size factor; β_{HML} is the beta for book-to-market tor; β_{WML} the beta momentum factor; SMB the premium for the size factor; HML is the premium for the fraction $\frac{VC}{VM}$ (book value over market value); and WML is the momentum effect.

Caldeira, Moura and Santos (2013, p.46) emphasized that "models of factors emerge as a promising alternative to solve the problem of dimensionality and offload the econometric estimation process”. The CAPM with FFC factors paved the way to facilitate the process of optimization of portfolios, finding ways to achieve higher returns and lower risks. For many years various types of optimized portfolios were tested in different countries, all being compared to the naïve portfolio. However, despite being superior in most tests, one of its characteristics prevented the equally weighted portfolio to be set aside, this feature is called turnover.

The turnover are all transactions of financial assets made by an investor, from purchase to selling them, generating costs on all drives. In any optimization process is demanded a greater number of transactions to be able to maintain a high level of return and low risk. Thus, transaction costs become problematic and may cause the chosen method to be undesirable.

In effort to reduce the turnover problem Kirby and Ostdiek (2012) propose two alternative methods: the Volatility Timing (VT) and Reward to Risk Timing (RRT). The VT is a method in which it was observed and corrected two characteristics - the risk that is brought from the estimation error of the expected returns and the high transaction costs from the portfolio optimization methods. Within this methodology "portfolios are rebalanced monthly based only on changes in the estimated conditional volatilities of asset returns" (KIRBY, OSTDIEK, 2012, p. 439). As seen in Maillard et al. (2008) the combination of the contributions of the Naïve and Minimum-Variance portfolios, since this last one works with a vector of “ones” instead of expected returns, can help lower the estimation risk and follow the path of the risk-based portfolios that Lee (2011) presented in his article.

This VT strategy uses only temporal variances of assets to calculate their weights within the portfolio. Despite having a simple formula, compared to other methods, the Volatility Timing overcame the innocent diversification, and managed to remain superior in transaction costs in the dataset tested by Kirby and Ostdiek (2012a). VT ignores information about expected returns, but gains in reduction of risk estimation.

However, the Reward to Risk Timing (RRT) maintains the mitigation of risk estimation, using only the variances of asset returns, but also incorporates significant information on returns in some of its methods. Using the four-factor model of Fama, French (1991, 1992) and Carhart (1997), RRT were proposed in two different analyses: one includes the mean of the asset returns and the other the average beta calculated by the number of factors used ($\bar{\beta}$), thus increasing information that enhances the methodology and does not bring the high estimation errors that comes from average returns. These two strategies, especially the RRT, succeeded in several studies in overcoming the $\frac{1}{N}$ method and also in controlling the turnover of their portfolios. It is important emphasize that these methods were not tested on the Brazilian market data before this study. Such models will be analyzed in more detail in the next section.

ANALYTICAL REFERENCE

For the analysis proposed in this research, the Brazilian stock market was represented by the group of assets that compose the Bovespa Index (Ibovespa), updated until December of 2014, for being a representative of the national stock market (BM&FBOVESPA, 2015), in a similar way as used in Iquiapaza at el. (2014). This theoretical portfolio is rebalanced along the year, so the assets available to form the portfolios were also updated accordingly. The number of the total assets that went through the portfolio was 113, and it was chosen only the securities that presented at least twenty full months continuously quoted with their price rates from January of 2004 to December of 2014. The assets present in this final sample were used in the construction of the Volatility Timing and Reward to Risk Timing strategies.

Kirby and Ostdiek (2012a) refer to four essential characteristics of both VT and RRT: (i) they do not use the inversion of the covariance matrix; (ii) they do not generate negative weights; (iii) they do not use optimization; and (iv) they adjust the sensitivity of the weights to the volatility changes with a *tuning* parameter η. See Fletcher (2011) for more information about the limited weight portfolios.

The VT strategy can be considerate as obtained by very simple calculations because it uses only two basic indicators in its formula: the variance and the parameter that determines the aggressiveness of the investor – representing how faster the investor will review their portfolios and rebalancing it when volatility changes.

The weights of VT method are obtained by the following formula:

$$\omega_{it} = \frac{(\frac{1}{\sigma_{it}^2})^\eta}{\sum_{i=1}^{N}(\frac{1}{\sigma_{it}^2})^\eta} \tag{8}$$

Where ω_i is the weight of asset *i*; σ_i^2 the variance of asset *i*; η the measures of the aggressiveness in which the investor balances the assets of its portfolio; *t* is the period in which the weights are being calculated.

As could be see, the VT strategy considers only the main diagonal of the covariance matrix (M), thus implicitly assuming that the expected pair-wise correlations are close to zero. According to Kirby and Ostdiek (2012a), the tuning parameter is a proxy for the change of the weights in relation to changes in the volatility of the assets in time. If $\eta = 0$ the result is the innocent portfolio 1/N; when η increases more weight is assigned to less volatile assets and vice versa; then if we set $\eta \rightarrow \infty$ the weight of the asset with less variance will tend to 1. In other words, if an asset's volatility increases the investor will reduce its weight more dramatically when η is greater. Thus, we follow the idea of $\eta > 1$, which will compensate for the loss of information by the not consideration of correlations between asset returns. For this study, it was considered η assuming the values {1, 2, and 4} for the comparison purpose.

The other strategy tested is the RRT, that comparatively is slightly more robust than the VT because try to not ignore the information about the expected returns. Kirby and Ostdiek (2012a) discuss the evolution of this strategy. On the assumption that the estimated pair-wise correlations between the excess risky-asset return are zero, the variance and the returns were used in the formula above to calculate the RRT using the average excess of returns of the assets (RRT).

$$\omega_{it} = \frac{(\hat{\mu}_{it}/\hat{\sigma}_{it}^2)^{\eta}}{\sum_{i=1}^{N}(\hat{\mu}_{it}/\hat{\sigma}_{it}^2)^{\eta}} \quad (9)$$

Where ω_{it} is the weight of the assets in the portfolio, $\hat{\mu}_{it}$ the mean of the excess of returns and $\hat{\sigma}_{it}^2$ is the variance from the asset i along the time t.

Kirby and Ostdiek (2012a) discussed that because the expected return are typically estimated with less precision than variances, the strategy is likely to entail significantly higher levels of estimation risk than the VT strategies, because of the extreme weights acquired if the means were to be negative for some assets, possibly causing the denominator to get close to zero.

To avoid this distortion, they proposed the RRT with the positive constraint (RRTK), where $\mu_{it} \geq 0$, which could be calculated by

$$\omega_{it} = \frac{(\hat{\mu}_{it}^{+}/\hat{\sigma}_{it}^2)^{\eta}}{\sum_{i=1}^{N}(\hat{\mu}_{it}^{+}/\hat{\sigma}_{it}^2)^{\eta}} \quad (10)$$

Where the $\hat{\mu}_{it}^{+}$ is the mean of the asset *i* conditioned to the positivity. Using only the positive average excess returns assume that the investor will not invest in assets with expected returns lower than the risk free asset.

It was also proposed an even more robust RRT strategy based on the four-factor model of the CAPM suggested by Fama and French (1992, 1993) and Carhart (1997). The RRT beta uses the average beta of the four factors found in a regression to substitute the mean of excess returns on the first type of RRT. It considers investors that assume that the factors are constant, and equal, so they calculate a simple mean of the four betas, but also that they will only use positive betas on their calculations. In this article the betas were tested by significance in p-value of 10% and then added two different types beta RRTs called RRTbm and RRTbms, where the *bm* is all the positive mean betas, and the *bms* are all the significant mean betas found in the regression of the CAPM 4-Factor.

$$\omega_{it} = \frac{(\bar{\beta}_{it}^{+}/\hat{\sigma}_{it}^2)^{\eta}}{\sum_{i=1}^{N}(\bar{\beta}_{it}^{+}/\hat{\sigma}_{it}^2)^{\eta}} \quad (11)$$

The $\bar{\beta}_{it}^{+}$ is the mean positive beta of the asset i conditioned to the positivity restriction in the period t.

All the strategies will be compared to the Naïve portfolio, Ibovespa, Minimum-Variance, and CDI (Interbank Deposit Certificate return). For the tests purpose, we also added a different type of Naïve portfolio that constraints the investor to only choose securities that have positive excess returns and we called it Wnp. As performance indicators for all the strategies we used the accumulated return on portfolio, average return on the portfolio, standard deviation, Sharpe Ratio, Herfindahl Index, turnover and the Breakeven as used by Santos and Tessari (2012) and Iquiapaza et al. (2014).

Furthermore, to check if there is different tendency along the whole period, the strategies will be analyzed according to three specific time periods: March 2009 through March 2011, April 2011 through December 2012, and January 2013 through December 2014.

The entire compilation of the databases and formulas for each strategy were modeled in R statistical software, from R Project for Statistical Computing, using 132 months and a rolling window equals to 60, resulting 72 windows with different assets weights that composed the Ibovespa index portfolio. CDI rates were used as a proxy for the risk free asset used on all calculations throughout the study. The returns and the asset prices were obtained from the Economatica database and BM&FBovespa from January of 2004 to December of 2014.

At first, the portfolios were formed, and then returns of the different VT and RRT strategies and its performance indicators were computed, over the rolling windows with their set of securities.

The table 1 summarizes all the basic information for the strategies used along this study.

Table 1 – Descriptive information of the strategies analyzed in the study.

Strategies	**Symbols**	**Description**
Naïve Portfolio	Wn and Wnp	Portfolio created with equal division between the assets. The Wnp has a restriction of allocation to only positive excess returns assets.
Minimum-Variance Portfolio	Wvm	Minimum-Variance portfolio built by Markowitz (1952) using rolling window forecasting.
Volatility Timing	VT1, VT2 and VT4	Strategy that uses only variance forecasting to calculate portfolio weights. The number in front of the symbol "VT" represents the value of the tuning parameter η used for the strategy.
Reward to Risk Timing with and without Positive excess returns restriction	RRT1, RRT2, RRT4, RRTk1, RRTk2, RRTk4	Strategy that adds more information to the VT, such as the mean excess returns for the RRT, and with the positive constraint for the excess of returns for RRTk. The number in front of each symbol also represents the tuning parameter η used for the strategy.
Reward to Risk Timing with Aver-	RRTbm2, RRTbm4,	These ads to the VT strategy information, the average positive Beta calculated from a re-

age Beta and Significant Average Betas	RRTbms2, RRTbms4	gression of the CAPM 4-Factor model in the case of the RRTbm, and restriction of only significant mean Betas for RRTbms. The number in front of each symbol also represents the tuning parameter η used for the strategy.
Ibovespa Index	IBov	A theoretical portfolio created for indexation of the BM&Fbovespa stock market in Brazil.
Risk Free Asset	CDI	The Interbank Deposit Certificate (CDI). This is a proxy for the risk free asset used in the article.

The Bovespa Index (Ibovespa) is a theoretical portfolio that indicates the average performance of the most negotiable assets of the BM&FBovespa. It also uses the assets that most represents the Brazilian market at a given time, therefore the assets that compose the Ibovespa portfolio are rebalanced periodically, changing its composition every trimester. Only the BM&FBovespa's securities can be part of this index and they have to follow a set of rules to be qualified as part of the portfolio (BM&FBOVESPA, 2014).

Along the analyzed period, some assets were included on Ibovespa, and in the study, considering the minimum permanency of twenty months in a row- that is one of the restrictions imposed on the dataset used. It is important to say that since the rebalancing of the Ibovespa is quarterly some of the portfolios had the same set of assets, but after each rebalancing those assets could change.

The next section will present the data characteristics and the results of the strategies applied to the Brazilian dataset.

RESULTS AND DISCUSSIONS

Along the study period some assets where more frequent in the Ibovespa - the benchmark for the analysis. The higher frequency of some assets could be explained by at least one of two characteristics: (i) a high participation in the Brazilian market because of the company size or the importance of that company to the Brazilian economy, or (ii) a great negotiability by the company during the process of composition of the index. Some examples are Petrobras (PETR3, PETR4) that holds an oil monopoly in Brazil, which gives it a high degree of representativeness in its national market even though the last years the company has been through one of its worse financial periods. Itaú Unibanco (ITSA4) in the other hand has had a great negotiability, which could indicate its high frequency inside the Ibovespa index.

In the case of the less frequent assets in the Index, we can say that they are less representative, did not showed liquidity during the rebalancing stages of the index or maybe they passed the criteria to be part of the portfolio because of an extraordinary event. From these, the assets from Companhia de Bebidas das Americas – AMBEV(ABEV3) one of the biggest beer enterprise, Redecard (RDCD3) and UNIBANCO (UBBR11) were the three which had the lowest frequency on the Index (Ibovespa).

After the dataset main characteristics, we present the results obtained by the analyzed strategies. Figure 1 shows the cumulated monthly returns of the strategies along the period of the 72 rolling windows. From that is visible that the portfolios followed a very close tendency of the market until March of 2011, at a point they started to differentiate. It is also visible that the Wvm, VT4 and RRT4 strategies had the highest returns since September of 2012.

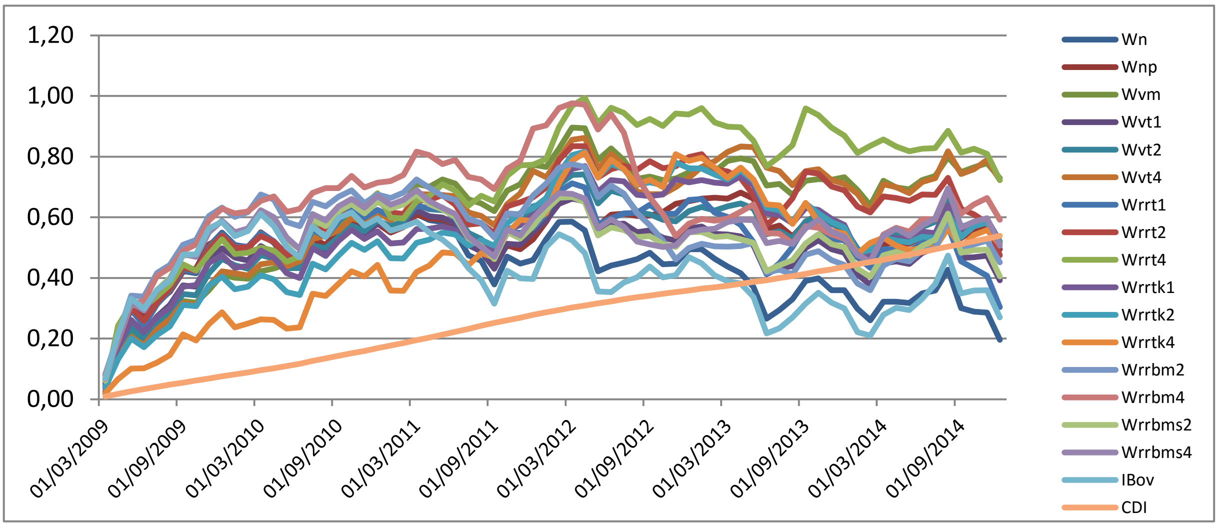

Figure 1 – The returns of portfolios formed following the VT and RRT strategies, with different investor’s aggressiveness (η).

Source: Elaborated by the authors with research results

After the overview of the results, we present a deeper analysis of the strategies. The Table 2 presents the descriptive indicators for the monthly returns of portfolios.

As presented in Table 2, all the strategies had a negative return as a minimum, where WRRTbm4 had the worse one with -14% in one of the 72 portfolios, followed by the Naïve, WRRTbms2 and WRRTbms4 with -13.48%, -12.85% and -12.91%, respectively.

The highest maximum return was presented by the unconstraint RRT4 with 20.34%, followed by the Naïve, RRT1 and RRT2 with 16.13%, 17.33% and 18.94%, in this order. Analyzing the mean, the lowest results were obtained by the Naïve and Ibov reaching 0.28% and 0.39%; in contrast, the highest returns were registered by the Minimum-variance (Wvm), VT4 and RRT4 with 1.04% and the other two with 1,03%, respectively.

Table 2 – Descriptive statistics for portfolios monthly returns – Minimum (Min), Median, Mean, Maximum (Max) and 1st quintile (1st Qu.) and 3rd quintile (3rd Qu.).

Indicators	Wn	Wnp	Wvm	Wvt1	Wvt2	Wvt4
Min.	-0,13480	-0,10670	-0,10460	-0,11180	-0,09770	-0,10100
1st Qu.	-0,03020	-0,01880	-0,01190	-0,02360	-0,01740	-0,01280
Median	0,00016	0,00610	0,00930	0,00190	0,00750	0,01150
Mean	0,00279	0,00710	0,01040	0,00560	0,00740	0,01030
3rd Qu.	0,03788	0,03410	0,04030	0,03990	0,03790	0,03720
Max.	0,16130	0,14710	0,11290	0,13460	0,11790	0,10100
Indicators	**Wrrt1**	**Wrrt2**	**Wrrt4**	**Wrrtk1**	**Wrrtk2**	**Wrrtk4**
Min.	-0,12200	-0,10650	-0,08600	-0,09458	-0,09002	-0,08492
1st Qu.	-0,02660	-0,02170	-0,02130	-0,02169	-0,01963	-0,02508
Median	-0,00090	0,00200	0,01053	0,00399	0,00858	0,00913
Mean	0,00430	0,00700	0,01033	0,00730	0,00739	0,00729
3rd Qu.	0,04210	0,04140	0,03917	0,03922	0,03870	0,03894
Max.	0,17330	0,18940	0,20337	0,11749	0,10389	0,11005
Indicators	**Wrrbm2**	**Wrrbm4**	**Wrrbms2**	**Wrrbms4**	**IBov**	**CID**
Min.	-0,09956	-0,14498	-0,12860	-0,12911	-0,12620	0,00480
1st Qu.	-0,02516	-0,01978	-0,02478	-0,02399	-0,03610	0,00690
Median	0,00709	0,01206	0,00250	0,00609	0,00110	0,00780
Mean	0,00645	0,00845	0,00578	0,00723	0,00390	0,00770
3rd Qu.	0,03751	0,03954	0,03864	0,03792	0,03680	0,00860
Max.	0,13047	0,12406	0,14309	0,13270	0,14450	0,01100

Source: Elaborated by the authors with research results

To evaluate the performance of the strategies and compare them to each other, some indicators were used: accumulated return on portfolio, average return on the portfolio, standard deviation, Sharpe Ratio, Herfindahl Index, turnover - as used by Santos and Tessari (2012) and Iquiapaza et al. (2014) - and the Breakeven. The results of financial performance are presented on Table 3.

Following the bad results seen before, the Ibov and Naïve presented the lowest portfolios cumulated returns with 10.15% and 16.67%. In the other hand the Wvm, VT4 and RRT4 in this order, had the highest results presenting returns of 94.97%, 92.13%, 87. 84%.

An interesting result is the very close standard deviation from all the strategies, from 4.1% to 5.35%, that is not very far from the 5.79% presented by the Ibovespa.

Related to the indexes calculated, only 4 strategies presented positive Sharpe Indexes the Wvm (6.6%), VT4 (5.98%), RRT4 (5.08%) and RRTbm4 (1.42%), opposing the negative results from the other portfolios.

Table 3–Financial indicators for portfolios monthly returns.

Indicators	Wn	Wnp	Wvm	Wvt1	Wvt2	Wvt4
Accumulated return	0,10150	0,50650	0,94970	0,36890	0,56810	0,92130
Mean return	0,00280	0,00710	0,01040	0,00560	0,00740	0,01030
St. Deviation	0,05350	0,04940	0,04100	0,04750	0,04470	0,04380
Beta	0,88730	0,77960	0,49580	0,76510	0,66050	0,51670
Sharpe R.	-0,09160	-0,01280	0,06610	-0,04400	-0,00600	0,05980
Inform. ratio	-0,06480	0,13520	0,15860	0,08020	0,11790	0,15200
Herfindahl	0,00000	0,02660	0,14600	0,00700	0,03440	0,17170
Turnover	0,01800	0,11560	0,09070	0,02590	0,03940	0,06550
Breakeven	0,10940	0,03090	0,05860	0,11890	0,09760	0,08120
Indicators	**Wrrt1**	**Wrrt2**	**Wrrt4**	**Wrrtk1**	**Wrrtk2**	**Wrrtk4**
Accumulated return	0,22860	0,46290	0,87840	0,54940	0,56400	0,54830
Mean return	0,00430	0,00680	0,01030	0,00730	0,00740	0,00730
St. Deviation	0,05330	0,05220	0,05170	0,04580	0,04460	0,04570
Beta	0,85020	0,79270	0,66820	0,67330	0,59740	0,46600
Sharpe R.	-0,06310	-0,01770	0,05080	-0,00860	-0,00700	-0,00900
Inform. ratio	0,02190	0,10560	0,16450	0,11290	0,09660	0,07130
Herfindahl	0,01270	0,05340	0,23480	0,06340	0,14170	0,35320
Turnover	0,10460	0,14300	0,18090	0,10800	0,12520	0,14480
Breakeven	0,02770	0,03150	0,03600	0,03150	0,03060	0,02990
Indicators	**Wrrbm2**	**Wrrbm4**	**Wrrbms2**	**Wrrbms4**	**IBov**	
Accumulated return	0,43390	0,63800	0,36610	0,50760	0,16670	
Mean return	0,00640	0,00840	0,00580	0,00720	0,00390	
St. Deviation	0,05130	0,05270	0,05170	0,05240	0,05790	
Beta	0,76600	0,60670	0,85400	0,82790	1,00000	
Sharpe R.	-0,02450	0,01420	-0,03710	-0,00900	-0,06640	
Inform. ratio	0,08930	0,10120	0,11100	0,14410	0,00000	
Herfindahl	0,04400	0,15840	0,01830	0,07550	NA	
Turnover	0,07470	0,11280	0,07380	0,11590	NA	
Breakeven	0,04050	0,03080	0,04140	0,03060	NA	

Source: Elaborated by the authors with research results

Another main result of this study was the analysis of turnover rates. Looking at that perspective the VT strategies went accordingly to what was expected, a turnover that followed closely the Naïve portfolio. The 1/N had a turnover of 1.8% and the VT (1), (2) and (4) had ratios of 2.6%, 3.9% and 6.5%, respectively, compared to a 9.07% of the Wvm portfolio.

The strategy RRT is analyzed aside because it did not follow the expected low turnover. The RRTs that had turnover under 10% were bm2 and bms2 with 7.5% and 7.4%, in this order. They also had worse performance than the VT, with negative Sharpe Ratios, being aside only the RRT4 and RRTbm4 in that indicator. In opposite, only the RRT4 had a high cumulated return with 87.84%. Although that last strategy had greater return, it did not solve the other main issue of a portfolio, which is the maintenance of a low turnover while achieving higher returns and lower risks.

Tables 4, 5 and 6 were designed to show the annualized returns, standard deviation, Sharpe Ratio and Worst Drawdown according to a different time period than the whole set of 132 months - not applying the rolling window. Looking for a better understanding of the strategies along the analyzed period, we separated it in three parts: March 2009 through March 2011, then April 2011 through December 2012, and finally January 2013 through December 2014.

Table-4 – Financial indicators for annualized returns, from March 2009 through March 2011

Indicators	**Wn**	**Wnp**	**Wvm**	**Wvt1**	**Wvt2**	**Wvt4**
Annualized Return	0,31580	0,31490	0,38970	0,32810	0,34280	0,36980
Annualized Sd	0,18300	0,17970	0,12170	0,15750	0,13940	0,11720
Annualized Sharpe (Rf=9.24%)	1,09670	1,11160	2,20780	1,34610	1,61810	2,13580
Indicators	**Wrrt1**	**Wrrt2**	**Wrrt4**	**Wrrtk1**	**Wrrtk2**	**Wrrtk4**
Annualized Return	0,33120	0,34160	0,33970	0,28910	0,26420	0,21010
Annualized Sd	0,18680	0,18940	0,19440	0,16500	0,15040	0,14030
Annualized Sharpe (Rf=9.24%)	1,14950	1,18420	1,14510	1,06730	1,01860	0,73750
Indicators	**Wrrbm2**	**Wrrbm4**	**Wrrbms2**	**Wrrbms4**	**IBov**	**CID**
Annualized Return	0,38960	0,45650	0,35630	0,36420	0,29630	0,09740
Annualized Sd	0,17490	0,14750	0,18480	0,18950	0,20100	0,00330
Annualized Sharpe (Rf=9.24%)	1,53510	2,23720	1,28710	1,29300	0,90860	0,00000

Source: Elaborated by the authors with research results

During the first sub period (Table 4), the strategies presented an increasing tendency, but they were all close together, in the second period (Table 5) the difference between their results started to increase and follow a decreasing tendency. Going into the third sub period (Table 6) most of the portfolios went into a convergence leaving 4 portfolios to be considered the best.

After the examination of the results, it was possible to conclude that the best strategies in an overall way were Wvm, VT4 and RRT4 in this order with cumulated returns of, 94.97%, 92.13%, 87.84%. They also had a relatively low risk. These three strategies also had the best Sharpe ratios.

We also compared the strategies with the performance of the Ibovespa portfolio and the growth of the CDI as the free risk asset. Thus, the article could show a representation of a market portfolio and verify how well it does and how efficient it is if an investor with lack of knowledge would come to use it. The index portfolio showed a low cumulated return with 16.67% and a negative Sharpe ratio, indicating a risky and inefficient method of investing during the specific Brazilian scenario.

Table-5 – Financial indicators for annualized returns, from April 2011 through December 2012

Indicators	Wn	Wnp	Wvm	Wvt1	Wvt2	Wvt4
Annualized Return	-0,07900	0,01680	0,01030	-0,04400	-0,02200	0,01680
Annualized Sd	0,18040	0,13790	0,15560	0,15140	0,14910	0,16760
Annualized Sharpe (Rf=9.24%)	-0,88800	-0,52100	-0,50100	-0,84300	-0,72200	-0,43000
Indicators	**Wrrt1**	**Wrrt2**	**Wrrt4**	**Wrrtk1**	**Wrrtk2**	**Wrrtk4**
Annualized Return	-0,00100	0,07400	0,16240	0,08230	0,13810	0,21330
Annualized Sd	0,16530	0,15370	0,15920	0,13470	0,14880	0,17470
Annualized Sharpe (Rf=9.24%)	-0,53500	-0,12500	0,38950	-0,08680	0,26650	0,62330
Indicators	**Wrrbm2**	**Wrrbm4**	**Wrrbms2**	**Wrrbms4**	**IBov**	**CID**
Annualized Return	-0,13350	-0,14610	-0,08010	-0,08770	-0,08300	0,09800
Annualized Sd	0,16620	0,20760	0,15560	0,15500	0,19660	0,00550
Annualized Sharpe (Rf=9.24%)	-1,26480	-1,06880	-1,03480	-1,08420	-0,83000	0,00000

Source: Elaborated by the authors with research results

Table-6 - Financial indicators for annualized returns, from January 2013 through December 2014

Indicators	Wn	Wnp	Wvm	Wvt1	Wvt2	Wvt4
Annualized Return	-0,15200	-0,09100	-0,01800	-0,09500	-0,06100	-0,01600
Annualized Sd	0,17350	0,17640	0,13200	0,16660	0,15970	0,15690
Annualized Sharpe (Rf=9.24%)	-1,31900	-0,97400	-0,79600	-1,05500	-0,90500	-0,65800
Indicators	**Wrrt1**	**Wrrt2**	**Wrrt4**	**Wrrtk1**	**Wrrtk2**	**Wrrtk4**
Annualized Return	-0,17600	-0,16300	-0,11400	-0,10840	-0,12520	-0,13860
Annualized Sd	0,17740	0,17370	0,16390	0,15860	0,14990	0,14620
Annualized Sharpe (Rf=9.24%)	-1,41300	-1,37500	-1,18100	-1,18810	-1,35960	-1,47760
Indicators	**Wrrbm2**	**Wrrbm4**	**Wrrbms2**	**Wrrbms4**	**IBov**	**CID**
Annualized Return	-0,03640	-0,00680	-0,08460	-0,03720	-0,11100	0,09390
Annualized Sd	0,16690	0,16430	0,17450	0,17870	0,19270	0,00460
Annualized Sharpe (Rf=9.24%)	-0,73270	-0,57840	-0,95460	-0,68860	-0,99100	0,00000

Source: Elaborated by the authors with research results

Related to the main question of this study, the results are inconclusive from the point of view of comparing strategies as a whole, but could indicate some very interesting results. All of the strategies presented very low Sharpe Ratios, which were mostly negative. Only four portfolios presented positive indexes such as 6.6% for Wvm, 5.98% for VT4, 5.08% for RRT4 and 1.42% for RRTbm4. These also had the highest portfolios returns ranging from 63.8% for the

RRTbm4 to 94.97% for the Wvm. However, the RRT portfolios presented high turnover, 18.09% for RRT4 and 11.28% for RRTbm4 while the VT4 features only 6.55%, which represent greater transaction costs.

Even though it was not possible to determine the better strategy, the Minimum-Variance strategy presented good overall results, result that was seen also in Iquiapaza at el. (2014), Sharma (2015), and Rubesam and Beltrame (2013), which means the difficulties in the determination of the better portfolio strategies. Rubesam and Beltrame (2013) advise that if the investor does not have a great method for the forecasting of the returns, this parameter should not be used. Hence volatility strategies like the used ones (VT and RRT) and the benchmarks such as Naïve and Minimum-Variance portfolios are recommended.

The results obtained give the investor the idea that the best choice of portfolio will depend upon the Brazilian economic scenario. During the first period RRTbm4 had the best results with 45.65% annualized return and a 223.7% Sharpe Ratio, the second period the RRTk4 featured with a 21.33% annualized return and a Sharpe Ration of 62.33%. In the last period, however, no strategies presented positive returns or Sharpe Ratios. A similar result was found by Sharma (2015), where no risk-based portfolio consistently outperforms the others under all market conditions.

The focus of this research was to show better portfolios that preserved a low turnover. As result, only two portfolios followed these requirements: the VT4 and the Wvm, which goes against what Kirby and Ostdiek (2012a) expected when they proposed them.

As said by DeMiguel et al (2009, p.1915) “there are still many “miles to go” before the gains promised by optimal portfolio choice can actually be realized out of sample” which shows that the focus on turnover, rather than optimization, when creating portfolio strategies for the stock market, is a promising field of research.

After all, it is important to say that Neto et al. (2008) criticize the use of Ibovespa as benchmark of the Brazilian stock market, but we choose it because the difficulty to find another index or investment fund that have a long temporal series and constant method of selection of its portfolio (BAIMA; COSTA, 2010).

It is very important to emphasize that these results were found according to the Brazilian economic and market characteristics, were all tendencies could be explained by the scenario of that specific time analyzed, which includes periods such as the economic boom during 2009 and 2010, and the economic collapse on the recent years with decreasing of GDP, increasing of the inflation rate and the devaluation of the exchange rates, which were not in the scope of this article and could be an good opportunity for later research.

CONCLUSIONS

This research aimed to verify the efficiency of the Volatility Timing (VT) and Reward to Risk Timing (RRT) strategies of portfolio selection proposed by Kirby and Ostdiek (2012a) applied to the Brazilian stock market, and then compared their results with the Naïve and Minimum-Variance portfolios.

After all the analyses, it was not possible to point out the best strategy, but should indicate relevant characteristics. Only four portfolios presented positive Sharpe indexes: In monthly terms Wvm (6.6%), VT4 (5.98%), RRT4 (5.08%) and RRTbm4 (1.42%). These also had the highest portfolios accumulated returns ranging from 63.8% for the RRTbm4 to 94.97% for the Wvm. However, the RRT portfolios presented high turnover, 18.09% for RRT4 and 11.28% for

RRTbm4 while the VT4 features only 6.55%. All of the strategies presented very low Sharpe Ratios, which were mostly negative.

The results obtained pointed out that the best choice of portfolio depends upon the Brazilian economic scenario. During the first period RRTbm4 had the best results with 45.65% annualized return and 223.7% Sharpe Ratio, the second period the RRTk4 featured with a 21.33% annualized return and a Sharpe Ratio of 62.33%. In the last period, however, no strategies presented positive returns or Sharpe Ratios.

Good overall results that were obtained by the Minimum-Variance portfolio can be seen in innumerous others researches, and as said by Rubesam and Beltrame (2013) this strategy is simple and easy to replicate, but also efficient when compared to the normal benchmark indexes such as CDI or Ibovespa.

Despite the good and important results obtained by this research about the portfolio selection and their qualities which enabled comparison of their indicators in the Brazilian reality, this research presents some limitations such as including only the assets that compose Ibovespa index and no considering the Macroeconomics variables that influence the financial market, being those great opportunities for future research.

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